Lunchtime at the Holy Purgatory

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Dramatis Personae

EWD

Roberte Recorde

Gottfried Leibniz

EWD   *Gottfried Leibniz is looking for E.W Dijkstra rather excitedly. Spots him sharing a table with Roberte Recorde and joins them for lunch.*

LEIBNIZ (*Turning to EWD*):   Edsgar! Remember, what you said once said?

> "If 10 years from now, when you are doing something quick and dirty, you suddenly visualize that I am looking over your shoulders and say to yourself, 'Dijkstra would not have liked this,' well that would be enough immortality for me."

So, you are immortal by your own definition!

DIJKSTRA (*with a glint in his eye*)   So are you Gottfried - your exposition on Purgatory got you a place here!

> *switching contexts*

So, you have also been following this Dot conversation?

> *Helps himself to yet another spoonful of steaming soup*

LEIBNIZ: Yes, of course!

> *Leibniz grabs some slabs of cheese - and begins with this sumptious helping*

Cheese at Purgatory is so much more gentle - almost like from the Leipzig dairy.

Dijkstra twitching his nose at the cheese with disgust

DIJKSTRA: (*returning to the conversation*)   They are struggling, but have not struck the right chord.

AUTHOR: Well, the chord is not vivifying enough... but you are kind of responsible for this flat-chord culture, aren't you? You made Leibniz' calculus as a style that pervades logic and programming - emphasizing calculational techniques at the level of form. And now, discussions are entirely at that level alone.

RECORDE   :It sometime chanceth, that the understanding most generally received, beeth not most deep. But because the understanding of that matter dependeth upon profounder knowledge than in that discussion could be uttered, we must not let it pass till some other time.

LEIBNIZ: Form is necessary - there is a point at which form should allowed to take over- to calculate.

AUTHOR: Today is a rather blessed day of cosmic alignments when I have you all together. My original question, Prof Dijkstra, was about why you introduced a symbol for function application - the Dot. Now, I realise that it sits in a bigger context of questions of *form, content and their interplay*. I have been granted very little time - I am on a limited visa to Purgatory and should be expelled from here if I don't take the vaporiser shuttle that leaves in 10 minutes. Sirs, please share with me your wisdom on these matters - and demolish the babel of our tongues...

DIJKSTRA (*responding to the urgency of time*):   Roberte, you invented the $=$. why?

RECORDE: To avoide the tediouse repetition of these woordes: *is equalle to*. I settled as I doe often in woorke use, a paire of paralleles, or gemowe (twin) lines of one lengthe: $=$, bicause noe 2 thynges, can be moare equalle.

DIJKSTRA: And you, Gottfried. What was the *Identity of Indiscernibles* that you published in the *Discourse on Metaphysics*?

LEIBNIZ: Well, it states that no two distinct substances exactly resemble each other and is typically understood to mean that no two objects have exactly the same properties. If, for every property F, object x has F if and only if object y has F, then x is identical to y. Or in the notation of symbolic logic:

$$\forall F(Fx <-> Fy) \to x = y$$

You are perhaps interested on the converse of this principle:

$$x = y \rightarrow \forall F(Fx < - > Fy)$$

I called this the Indiscernibility of Identicals.

DIJKSTRA: Yes, I have christened it the Leibniz's Rule. So let me tell you what I did - what you called *property of x* and denote as *Fx*, is *the value of a function F at x*. So your rule says:

$$x = y \rightarrow Fx = Fy$$

and links up the sameness of x and y with the value of the function F at x and y.

LEIBNIZ (*with surprise*) Did you say *function*? I used that word for the first time in a letter to Jean (Bernoulli) in 1673. Also *variables, constants, terms*etc.

DIJKSTRA You see, 150 years after you, Euler came with a new notation for functions and their application. And your rule got absorbed in the most general context of functions as

$$x = y \rightarrow f(x) = f(y)$$

Roberte, given the context in which you invented =, what would you do to avoid the tediouse repetion of the words *value of function at* which occurs again and again in all sameness preserving transformations described by the Leibniz rule

RECORDE (*reflectively*) It needeth a symbol. Euler'symbol ()? What about that?

LEIBNIZ: You want to represent the relation of application between a function and its argument. Remember *Dialogue on the Connections Between Things and Words* circa 1677? When inventing a new form to represent a relation or an object, it better be a single word.
Edsgar! not a good idea to make that relation in 2 symbol-parts separated by the argument variable as (x).

DIJKSTRA Thats what I did. The parenthesis were quite overloaded with many meanings, by the time I had found my place of dwelling on the earth. Also, while there were symbols to describe application, the sense of application being a single operation or a relation beween the function and its argument was lost. So, I introduced the single Dot, so that the Leibniz rule now reads as

$$x = y \rightarrow f.x = f.y$$

In fact, strictly speaking it is

$$x = y \rightarrow \forall f f.x = f.y$$

In other words, it says, that *function application perserves sameness* – since it is true of any f, quite independent of f, it describes the relation between = and .

AUTHOR: Ah, I see. It has a deep semantic import... its amazing how you you guys shift from form to content and backwards - packing so much content into form and then calculating purely with form to find new meanings...

DIJKSTRA: (*still relishing his soup*) Aye Aye.. Thats how it goes. Language does not throw away meaning, but one cannot discuss the etymolgy of each word, every time its uttered...

Pointing to the vaporiser exit room as a strong stink pervades the room

Thats your call, the vaporising escalator bound for earth is about to leave. That smell is like a bell. Rush, if you miss it you might get thrown out of here and will hang between Purgatory and the Earth...err between Content and Form

And the author scrambles for the door.

Enters the door of the "Inter-content-former" –an escalator that has been designed to shuttle between Purgatory and the Earth. The Arch-engineer who has designed this system is visiting for an inspection to investigate why there are fewer and fewer earth beings vising purgatory these days. The author recognises him as soon as he enters

AUTHOR Oh God! Thats Church! (*As if pouncing on him with a question*)Only you...

CHURCH (*unmindful of his comment*) Welcome aboard, son! I'm glad that you have completed your interplanet sojourn. But what is it that causes so much strain to you? You look confused!

AUTHOR I am caught in the pull between form and content. On earth, all meaning is intermeshed into the fabric of form. On Purgatory, they co-exist in a concious divide – much effort to represent content in form and then one lets the symbols work. Something is still amiss...

Church   You must be a programmer! (*with a wink*) What do they call them nowadays? Functional programmers?

Author (*Not letting go of his question*)   The form-content separation and the program-data divide – I wonder what is their relation.

Church   Hmnnn... Answers to all divides can only lie in that transit zone – when *form circulates to become content.* Or should I say *dot becomes the argument to dot?*

> *As the door opens to place him on the threshold between form and content*

Time for you to disembark – (*smiling*) transit zones are always brief!

> *Before he can even notice he is out, the door shuts leaving behind a residual mysterious taste: where in the history of functions and notations does Church belong?*